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MOTIVE FORCES IN A COMPLEX MODEL INVOLVING A DIRECT FLOW AND MIXING OF A LIQUID

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The influence of liquid mixing on mean logarithmic and arithmetic forces as well as the numbers of transfer units in direct flow has been studied. The extreme cases where liquid on the plate is entirely mixed and where it moves in a regime of perfect displacement are considered. The interrelationship between the mean arithmetic forces and numbers of transfer units in the case of complete liquid mixing and without it has been found. Dependences of the efficiency on the numbers of transfer units are determined.

The effectiveness of mass transfer in direct motion of interacting flows is independent of liquid mixing on the tray. This characteristic feature of the direct flow is predetermined by the conditions of equilibrium between the vapor and liquid leaving the ideal plate and is observed in a complex model [1] and variants of mass transfer [2] with conditions of the relationship between an ideal plate and a real one that are typical of the models of Murphree and Hausen. In the most widespread models of Murphree [3–5] and Hausen [4–6], only complete mixing of liquid is foreseen. The computational relations, in particular, the effectiveness of mass transfer, are identical in the variants of mass transfer without mixing and in the above-mentioned models.

At the same time, mixing of liquid exerts its influence on the magnitude of the motive forces. A liquid flow of composition x_n arriving at the plate mixes up with the liquid present there and partially mixed and its concentration is decreased to x_{in} (see Fig. 1). As a result of interaction with the vapor, the liquid is depleted of the highly volatile component and leaves the contact stage with the concentration x_{n-1}.

In [7, 8], with countercurrent and crosscurrent motion of phases, the mixing of the liquid is taken into account by the amount of completely mixed liquid φ. The other part of the liquid $(1 - \varphi)$ moves on the plate in the regime of perfect displacement. Thus, the quantity φ characterizes the degree of liquid mixing. We will extend this expression to a direct flow. According to the assumption made, the composition of the liquid that arrives at the plate is expressed as

$$x_{in} = (1 - \varphi)\, x_n + \varphi x_{n-1} \, , \tag{1}$$

and of that leaving the plate as

$$x_f = x_{n-1} \, . \tag{2}$$

In a complex model with a direct flow [1], the initial composition of the arriving liquid, subject to formula (25) of [9], is equal to

$$x_n = x_{n-1} + \frac{(m+1)\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)E_{d.f}}{\dfrac{L}{V} + m + \dfrac{L}{mV}E_{d.f} - mE_{d.f}} \, , \tag{3}$$

and the concentration of the escaping vapor, subject to the material balance equation $L(x_n - x_{n-1}) = V(y_n - y_{n-1})$, is

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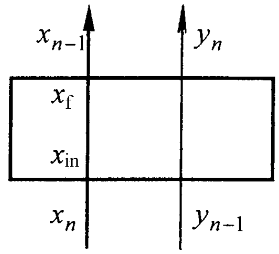

Fig. 1. Change in the concentration on the plate in direct flow.

$$y_n = y_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\mathrm{d.f}}}{\frac{L}{V} + m + \frac{L}{mV}E_{\mathrm{d.f}} - mE_{\mathrm{d.f}}}. \tag{4}$$

The initial and final (in the direction of the flows) motive forces, subject to (1)–(4), are the following:

$$\Delta x_{\mathrm{in}} = x_{\mathrm{in}} - \frac{y_{n-1}}{m} = \frac{\frac{L}{V} + m + E_{\mathrm{d.f}}\left[\frac{L}{mV} + 1 - \varphi\,(m+1)\right]}{m+1}\,\frac{x_n - x_{n-1}}{E_{\mathrm{d.f}}}, \tag{5}$$

$$\Delta x_{\mathrm{f}} = x_{\mathrm{f}} - \frac{y_n}{m} = \frac{\left(\frac{L}{V} + m\right)(1 - E_{\mathrm{d.f}})}{m+1}\,\frac{x_n - x_{n-1}}{E_{\mathrm{d.f}}}. \tag{6}$$

The mean logarithmic motive forces, expressed in terms of the parameters of the liquid and vapor phases, are equal respectively to

$$\Delta x_{\mathrm{m},\varphi,\log} = \frac{\Delta x_{\mathrm{in}} - \Delta x_{\mathrm{f}}}{\ln\frac{\Delta x_{\mathrm{in}}}{\Delta x_{\mathrm{f}}}} = \frac{\left(\frac{L}{mV} + 1 - \varphi\right)(x_n - x_{n-1})}{\ln\dfrac{1 + \frac{E_{\mathrm{d.f}}}{n} - \varphi\,\frac{m+1}{\frac{L}{V} + m}E_{\mathrm{d.f}}}{1 - E_{\mathrm{d.f}}}}, \tag{7}$$

$$\Delta y_{\mathrm{m},\varphi,\log} = \frac{\left[1 + \frac{mV}{L}(1 - \varphi)\right](y_n - y_{n-1})}{\ln\dfrac{1 + \frac{E_{\mathrm{d.f}}}{m} - \varphi\,\frac{m+1}{\frac{L}{V} + m}E_{\mathrm{d.f}}}{1 - E_{\mathrm{d.f}}}}. \tag{8}$$

In [10], replacement of mean logarithmic values by arithmetic means is admitted provided that the ratio of the larger motive force to the smaller one does not exceed two. As applied to liquid mixing, this condition is formalized in the form

$$\frac{1 + \frac{E_{\mathrm{d.f}}}{m} - \varphi\,\frac{m+1}{\frac{L}{V} + m}E_{\mathrm{d.f}}}{1 - E_{\mathrm{d.f}}} \le 2, \tag{9}$$

whence

$$E_{\text{d.f}} \le \frac{m}{2m + 1 - \varphi \dfrac{m+1}{\dfrac{L}{mV} + 1}} . \qquad (10)$$

With the liquid being mixed completely or in the absence of mixing, condition (10) is correspondingly simplified to

$$E_{\text{d.f}} \le \frac{1}{1 + \dfrac{m+1}{m\left(1 + \dfrac{mV}{L}\right)}} , \qquad (11)$$

$$E_{\text{d.f}} \le \frac{m}{2m+1} . \qquad (12)$$

With limitation (10) being satisfied, the corresponding mean arithmetic motive forces in liquid and vapor media are defined as

$$\Delta x_{\text{m},\varphi,\text{a}} = \frac{\Delta x_{\text{in}} + \Delta x_{\text{f}}}{2} = \left[\frac{\dfrac{L}{mV} + 1}{m+1}\left(\frac{m}{E_{\text{d.f}}} - \frac{m-1}{2}\right) - \frac{\varphi}{2}\right](x_n - x_{n-1}) , \qquad (13)$$

$$\Delta y_{\text{m},\varphi,\text{a}} = \left[\frac{1 + \dfrac{mV}{L}}{m+1}\left(\frac{m}{E_{\text{d.f}}} - \frac{m-1}{2}\right) - \frac{\varphi}{2}\frac{mV}{L}\right](y_n - y_{n-1}) . \qquad (14)$$

The dependences of the motive forces for the extreme cases of the direct flow where the liquid is completely mixed ($\varphi = 1$) and where mixing is absent ($\varphi = 0$) are presented in Table 1.

The expressions for the mean motive forces with complete mixing of the liquid from Table 1 coincide with formulas (6) and (9) of [10], and in the absence of mixing they coincide with certain dependences of [11] obtained in the case where there is perfect displacement of the liquid, which confirms the validity of the calculations performed.

The corresponding numbers of transfer units in the case where there is partial mixing of the liquid have the form

$$N_{\text{liq},\varphi,\log} = \frac{\ln \dfrac{1 + \dfrac{E_{\text{d.f}}}{m} - \varphi \dfrac{m+1}{\dfrac{L}{V} + m} E_{\text{d.f}}}{1 - E_{\text{d.f}}}}{\dfrac{L}{mV} + 1 - \varphi} , \qquad (15)$$

$$N_{\text{v},\varphi,\log} = \frac{\ln \dfrac{1 + \dfrac{E_{\text{d.f}}}{m} - \varphi \dfrac{m+1}{\dfrac{L}{V} + m} E_{\text{d.f}}}{1 - E_{\text{d.f}}}}{1 + \dfrac{mV}{L}(1 - \varphi)} , \qquad (16)$$

$$N_{\text{liq},\varphi,\text{a}} = \frac{1}{\dfrac{\dfrac{L}{mV} + 1}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right) - \dfrac{\varphi}{2}} , \qquad (17)$$

TABLE 1. Limiting Values of the Motive Forces in Direct Flow

Quantity	Complete mixing of liquid	Absence of mixing
$\dfrac{\Delta x_{\text{m,log}}}{x_n - x_{n-1}}$	$\dfrac{\dfrac{L}{mV}}{\ln \dfrac{1+\dfrac{E_{\text{d.f}}}{m} - \dfrac{m+1}{\dfrac{L}{V}+m} E_{\text{d.f}}}{1 - E_{\text{d.f}}}}$	$\dfrac{\dfrac{L}{mV}+1}{\ln \dfrac{1+\dfrac{E_{\text{d.f}}}{m}}{1 - E_{\text{d.f}}}}$
$\dfrac{\Delta y_{\text{m,log}}}{y_n - y_{n-1}}$	$\dfrac{1}{\ln \dfrac{1+\dfrac{E_{\text{d.f}}}{m} - \dfrac{m+1}{\dfrac{L}{V}+m} E_{\text{d.f}}}{1 - E_{\text{d.f}}}}$	$\dfrac{1+\dfrac{mV}{L}}{\ln \dfrac{1+\dfrac{E_{\text{d.f}}}{m}}{1 - E_{\text{d.f}}}}$
$\dfrac{\Delta x_{\text{m,a}}}{x_n - x_{n-1}}$	$\dfrac{\dfrac{L}{mV}+1}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right) - \dfrac{1}{2}$	$\dfrac{\dfrac{L}{mV}+1}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right)$
$\dfrac{\Delta y_{\text{m,a}}}{y_n - y_{n-1}}$	$\dfrac{1+\dfrac{mV}{L}}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right) - \dfrac{mV}{2L}$	$\dfrac{1+\dfrac{mV}{L}}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right)$

$$N_{\text{v},\varphi,\text{a}} = \frac{1}{\dfrac{1+\dfrac{mV}{L}}{m+1}\left(\dfrac{m}{E_{\text{d.f}}} - \dfrac{m-1}{2}\right) - \dfrac{\varphi}{2}\dfrac{mV}{L}} . \tag{18}$$

The boundary values of N for $\varphi = 1$ and $\varphi = 0$ are listed in Table 2, analysis of the data of which shows the identity between the dependences and the corresponding quantities of [10] and [11].

We will express the effectiveness of mass transfer in terms of $N_{\text{liq},\varphi,\text{a}}$ from formula (17):

$$E_{\text{d.f}} = \frac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\left(\dfrac{1}{N_{\text{liq},\varphi,\text{a}}} + \dfrac{\varphi}{2}\right) + \dfrac{m-1}{2m}} \tag{19}$$

and in terms of the number of transfer units in the absence of mixing from Table 2:

$$E_{\text{d.f}} = \frac{1}{\dfrac{m+1}{N_{\text{liq},\varphi,\text{a}}\left(\dfrac{L}{V}+m\right)} + \dfrac{m-1}{2m}} . \tag{20}$$

Since the effectiveness of mass transfer in direct flow is independent of liquid mixing, this makes it possible to equate the right-hand sides of (19) and (20). As a result, we find the relationship between the number of transfer units in the case of liquid mixing and in its absence:

$$N_{\text{liq},\varphi,\text{a}} = \frac{N_{\text{liq,a}}}{1 - \varphi\dfrac{N_{\text{liq,a}}}{2}} . \tag{21}$$

From expression (21) it follows that in order to attain the same effectiveness of mass transfer, the number of transfer units in mixing must be higher than the corresponding value without liquid mixing. In the absence of mixing, the numbers of transfer units are equalized, according to (21), and on complete mixing of the liquid they are equal to

TABLE 2. Limiting Values of V in Direct Flow

Number of transfer units	Complete mixing of liquid	Absence of mixing
$N_{\text{liq,log}}$	$\dfrac{\ln\dfrac{1+\dfrac{E_{\text{d.f}}}{m}-\dfrac{m+1}{\dfrac{L}{V}+m}E_{\text{d.f}}}{1-E_{\text{d.f}}}}{\dfrac{L}{mV}}$	$\dfrac{\ln\dfrac{1+\dfrac{E_{\text{d.f}}}{m}}{1-E_{\text{d.f}}}}{\dfrac{L}{mV}+1}$
$N_{\text{v,log}}$	$\ln\dfrac{1+\dfrac{E_{\text{d.f}}}{m}-\dfrac{m+1}{\dfrac{L}{V}+m}E_{\text{d.f}}}{1-E_{\text{d.f}}}$	$\dfrac{\ln\dfrac{1+\dfrac{E_{\text{d.f}}}{m}}{1-E_{\text{d.f}}}}{1+\dfrac{mV}{L}}$
$N_{\text{liq,a}}$	$\dfrac{1}{\dfrac{\dfrac{L}{mV}+1}{m+1}\left(\dfrac{m}{E_{\text{d.f}}}-\dfrac{m-1}{2}\right)-\dfrac{1}{2}}$	$\dfrac{m+1}{\left(\dfrac{L}{mV}+1\right)\left(\dfrac{m}{E_{\text{d.f}}}-\dfrac{m-1}{2}\right)}$
$N_{\text{v,a}}$	$\dfrac{1}{\dfrac{1+\dfrac{mV}{L}}{m+1}\left(\dfrac{m}{E_{\text{d.f}}}-\dfrac{m-1}{2}\right)-\dfrac{mV}{2L}}$	$\dfrac{m+1}{\left(1+\dfrac{mV}{L}\right)\left(\dfrac{m}{E_{\text{d.f}}}-\dfrac{m-1}{2}\right)}$

$$N_{\text{liq,c.mix,a}}=\frac{N_{\text{liq,a}}}{1-\dfrac{N_{\text{liq,a}}}{2}}, \tag{22}$$

which coincides with the data of [10] and confirms the calculations made.

From expression (13) we find the value of the efficiency of liquid mixing:

$$E_{\text{d.f}}=\frac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\left(\dfrac{\Delta x_{\text{m},\varphi,\text{a}}}{x_n-x_{n-1}}+\dfrac{\varphi}{2}\right)+\dfrac{m-1}{2}}, \tag{23}$$

the simplification of which on perfect displacement of the liquid ($\varphi = 0$) yields

$$E_{\text{d.f}}=\frac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\dfrac{\Delta x_{\text{m,à}}}{x_n-x_{n-1}}+\dfrac{m-1}{2}}. \tag{24}$$

Equating the right-hand sides because of the equality of the left-hand sides leads to the dependence

$$\Delta x_{\text{m},\varphi,\text{a}}=\Delta x_{\text{m,a}}-\varphi\,\frac{x_n-x_{n-1}}{2} \tag{25}$$

and to its modification for the case of complete mixing of the liquid:

$$\Delta x_{\text{m,c.mix,a}}=\Delta x_{\text{m,a}}-\frac{x_n-x_{n-1}}{2}. \tag{26}$$

TABLE 3. Limiting Dependences of E on N in Direct Flow

Efficiency	Complete mixing of liquid	Absence of mixing
$E(N_{\text{liq,log}})$	$\dfrac{\exp\left(\dfrac{L}{mV}N_{\text{liq,c.mix,log}}\right)-1}{\exp\left(\dfrac{L}{mV}N_{\text{liq,c.mix,log}}\right)+\dfrac{1}{m}-\dfrac{m+1}{\dfrac{L}{V}+m}}$	$\dfrac{\exp\left[\left(\dfrac{L}{mV}+1\right)N_{\text{liq,log}}\right]-1}{\exp\left[\left(\dfrac{L}{mV}+1\right)N_{\text{liq,log}}\right]+\dfrac{1}{m}}$
$E(N_{\text{v,log}})$	$\dfrac{\exp N_{\text{v,c.mix,log}}-1}{\exp N_{\text{v,c.mix,log}}+\dfrac{1}{m}-\dfrac{m+1}{\dfrac{L}{V}+m}}$	$\dfrac{\exp\left[\left(1+\dfrac{mV}{L}\right)N_{\text{v,log}}\right]-1}{\exp\left[\left(1+\dfrac{mV}{L}\right)N_{\text{v,log}}\right]+\dfrac{1}{m}}$
$E(N_{\text{liq,a}})$	$\dfrac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\left(\dfrac{1}{N_{\text{liq,c.mix,a}}}+\dfrac{1}{2}\right)+\dfrac{m-1}{2m}}$	$\dfrac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\dfrac{1}{N_{\text{liq,a}}}+\dfrac{m-1}{2m}}$
$E(N_{\text{v,a}})$	$\dfrac{1}{\dfrac{m+1}{\dfrac{L}{V}+m}\left(\dfrac{L}{mV}\dfrac{1}{N_{\text{v,c.mix,a}}}+\dfrac{1}{2}\right)+\dfrac{m-1}{2m}}$	$\dfrac{1}{\dfrac{m+1}{1+\dfrac{mV}{L}}\dfrac{1}{mN_{\text{v,a}}}+\dfrac{m-1}{2m}}$

It is seen from formulas (25) and (26) that the mean motive forces become smaller on mixing of the liquid:

$$E_{\text{d.f}}=\frac{\exp\left[\left(\dfrac{L}{mV}+1-\varphi\right)N_{\text{liq},\varphi,\text{log}}\right]-1}{\exp\left[\left(\dfrac{L}{mV}+1-\varphi\right)N_{\text{liq},\varphi,\text{log}}\right]+\dfrac{1}{m}-\varphi\dfrac{m+1}{\dfrac{L}{V}+m}}, \tag{27}$$

$$E_{\text{d.f}}=\frac{\exp\left[\left(1+\dfrac{mV}{L}(1-\varphi)\right)N_{\text{v},\varphi,\text{log}}\right]-1}{\exp\left[\left(1+\dfrac{mV}{L}(1-\varphi)\right)N_{\text{v},\varphi,\text{log}}\right]+\dfrac{1}{m}-\varphi\dfrac{m+1}{\dfrac{L}{V}+m}}. \tag{28}$$

The limiting efficiencies relating to the regimes of perfect mixing and displacement of the liquid on the plate are presented in Table 3.

The data obtained show in which way the mixing of the liquid influences the magnitudes of the motive forces and finally mass transfer in direct flow of a vapor and a liquid phase.

NOTATION

E, effectiveness of mass transfer; L, molar flow of liquid; m, coefficient of phase equilibrium; N, number of transfer units; V, molar vapor flux; x and y, concentration of a volatile component in the liquid and vapor phase, respectively; Δ, difference of concentrations; φ, quantity of completely mixed liquid (degree of liquid mixing). Subscripts: a, arithmetic value; φ, account for liquid mixing; f, final value; log, logarithmic value; liq, liquid phase; in, initial value; n, number of the plate considered; $n-1$, number of the previous upstream plate; d.f, direct flow; c.mix, complete mixing of liquid; m, mean value; v, vapor phase.

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